

## Cluster structures in stable and unstable nuclei\*

Yoshiko Kanada-En'yo,<sup>1,†</sup> Masaaki Kimura,<sup>2</sup> Fumiharu Kobayashi,<sup>3</sup>  
Tadahiro Suhara,<sup>4</sup> Yasutaka Taniguchi,<sup>5</sup> and Yuta Yoshida<sup>1</sup><sup>1</sup>Department of Physics, Kyoto University, Kyoto 606-8502, Japan<sup>2</sup>Department of Physics, Hokkaido University, Sapporo, 060-0810, Japan<sup>3</sup>Department of Physics, Niigata University, Niigata 950-2181, Japan<sup>4</sup>Matsue College of Technology, Matsue 690-8518, Japan<sup>5</sup>Nihon Institute of Medical Science, Moroyama-machi, Iruma-gun, Saitama 350-0435, Japan

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Cluster structures in light unstable nuclei are discussed. The structures of neutron-rich Be isotopes are theoretically investigated and the molecular orbital bond structure and its role in the vanishing of the neutron magic number  $N = 8$  are discussed. The two-body cluster resonances in highly excited states of neutron-rich Li, Be and B isotopes are predicted theoretically.

Keywords: Cluster, Molecular dynamics, Unstable nuclei

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## I. INTRODUCTION

Historically, many cluster structures have been discovered in light stable nuclei. More recently, various cluster structures have also been reported in the  $sd$ -shell and  $pf$ -shell regions of heavier nuclei and in unstable nuclei ([1–4] and references therein). These findings indicate that cluster structures are common over a wide region of the nuclear chart. If there is no correlation between nucleons, all nucleons in a nucleus behave as independent particles in a mean field. However, in reality, because of the attractive nuclear force, the correlation between nucleons occurs to form cluster cores at the nuclear surface. This is the cluster core formation and regarded as a kind of ground state correlation. In the cluster formation at the nuclear surface, clusters largely overlap with the core nucleus and the system still in a normal density state. In the system with cluster cores, intercluster motion is easily activated by a small amount of energy. Then, the cluster structures are spatially developed in excited states. This means that the mean-field and cluster states coexist in the low-energy regions of nuclear systems.

$^{12}\text{C}$  is a typical example of coexisting cluster and mean-field features. The ground state of  $^{12}\text{C}$  is the mean-field state dominated by the  $p_{3/2}$ -shell closed configuration mixed with the  $3\alpha$  cluster core structure. At around 100 MeV, all twelve nucleons in the  $^{12}\text{C}$  nucleus can dissociate, and the system evolves to a free nucleon gas state. At the low energy region around 10 MeV, three  $\alpha$  clusters develop spatially in excited states of  $^{12}\text{C}$ . The energy of the  $3\alpha$  cluster excitation is much smaller than that of the nucleon gas state, implying that the mean-field and cluster states coexist in the low-energy levels of  $^{12}\text{C}$ .

Recent studies have revealed further rich cluster phenomena also in unstable nuclei, in which valence nucleons play im-

portant roles. When excess neutrons are added to the already-clustered stable nuclei, the cluster structure weakens in some cases. If the additional neutrons deform the neutron structure the cluster structure can be further developed in neutron-rich nuclei. In neutron-rich Be and Ne isotopes, the cluster development is accompanied by the vanishing of the neutron magic number. Moreover, in remarkably developed cluster structures in Be and B isotopes, a new types of cluster structure called molecular orbital structure has been attributed to the valence neutrons in the molecular orbitals surrounding the  $2\alpha$  and  $^{16}\text{O}+\alpha$  cluster cores, respectively.

Furthermore, recent experimental and theoretical studies have revealed new states of cluster resonances containing exotic clusters in the highly excited states of various unstable nuclei such as He+He cluster states in Be isotopes [2, 3, 5–23],  $^{10}\text{Be}+\alpha$  states in  $^{14}\text{C}$  [24–28],  $^{14}\text{C}+\alpha$  states in  $^{18}\text{O}$  and their mirror states [29–38],  $^{18}\text{O}+\alpha$  states in  $^{22}\text{Ne}$  [36–43],  $^9\text{Li}+^6\text{He}$  states in  $^{15}\text{B}$  [12], and  $^6\text{He}+t$  states in  $^9\text{Li}$  [44].

Cluster structures have also been reported in heavier mass nuclei in the  $sd$ -shell and  $pf$ -shell regions. Examples are  $^{28}\text{Si}+\alpha$ ,  $^{24}\text{Mg}+\alpha$ ,  $^{28}\text{Si}+\alpha$ ,  $^{36}\text{Ar}+\alpha$ , and  $^{40}\text{Ca}+\alpha$  cluster states in  $^{28}\text{Si}$ ,  $^{32}\text{S}$ ,  $^{40}\text{Ca}$ , and  $^{44}\text{Ti}$ , respectively. These cluster states may coexist with different cluster channels such as  $^{16}\text{O}+^{12}\text{C}$ ,  $^{16}\text{O}+^{16}\text{O}$ ,  $^{28}\text{Si}+^{12}\text{C}$  and  $^{28}\text{Si}+^{16}\text{O}$  cluster structures in each nucleus. These facts indicate that various cluster structures appear over a wide region of the nuclear chart.

By theoretically investigating these cluster phenomena, we aim to acquire a systematic understanding of nuclear systems and investigate cluster phenomena in light nuclei with the antisymmetrized molecular dynamics (AMD) method [3, 45]. The AMD model describes both the cluster and mean field structures in general nuclei. One of the advantages of the AMD model is that the cluster formation and breaking, as well as the cluster excitation, can be described in the AMD framework without assuming the existence of any clusters. The AMD method is further explained in Ref. [3] and the references therein.

This paper is organized as follows. Section II discusses the cluster structures of Be isotopes obtained from the AMD

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† Corresponding author, [yenyo@ruby.scphys.kyoto-u.ac.jp](mailto:yenyo@ruby.scphys.kyoto-u.ac.jp)

calculation. The cluster resonances are discussed in Section III. The paper concludes with a summary in Section IV.

## II. CLUSTER STRUCTURES OF BE ISOTOPES

In Be isotopes, two  $\alpha$  clusters are formed even in the low-lying levels. In the case of  $^{10}\text{Be}$ , the ground state is the normal state having a  $2\alpha$  cluster core structure. In the excited state, the molecular orbital (MO) structure appears in the  $0_2^+$  state at 6.18 MeV, in which valence neutrons occupy the longitudinal molecular orbital,  $\sigma$  orbital, around the  $2\alpha$  core. The  $0_2^+$  state is the largely deformed state with the developed cluster structure, and it constructs a rotational band. The candidates of the band members, a  $2^+$  state at 7.54 MeV and a  $4^+$  state at 10.2 MeV, have been reported experimentally [19, 20]. We call this MO structure in the  $0_2^+$  state the MO bond structure because two  $\alpha$  clusters are bonded by valence neutrons in the MO around the  $2\alpha$  core. Very recently,  $^6\text{He}+\alpha$  cluster resonances have also been reported at around  $E_x=10$  MeV, a slightly higher energy than that of the MO bond.

The cluster features of the MO bond structure and those of the cluster resonance differ from each other. In the MO bond structure, two valence neutrons move throughout the system around  $2\alpha$ s. By contrast, two valence neutrons in the  $^6\text{He}+\alpha$  cluster resonance are localized around one of the two  $\alpha$ s to form the  $^6\text{He}$  cluster which weakly couples to the other  $\alpha$  cluster. Thus, two kinds of cluster structure appear in neutron-rich Be isotopes. One is the MO bond structure, and the other is the cluster resonance. The former is a strong coupling cluster structure, and the other is a weak coupling cluster structure. Similar cluster structures have also been reported in *sd*-shell nuclei such as  $^{22}\text{Ne}$ , for which the MO bond structure with the  $^{16}\text{O}+\alpha$  cluster core and the  $^{18}\text{O}+\alpha$  cluster resonances were predicted in excited states.

The picture of the MO structure proposed by Seya *et al.* and von Oertzen *et al.* well describes the cluster structures of low-lying states of Be isotopes [5, 6], and it is useful to understand the vanishing of the neutron magic number  $N = 8$  in neutron-rich Be. In the neutron-rich Be, the many-body correlation leads to the formation of two  $\alpha$  cluster cores. In the  $2\alpha$  system, MOs of a normal  $\pi$ -type orbital and a higher nodal  $\sigma$  orbital are constructed by the linear combination of the *p*-orbit around each  $\alpha$  cluster, and they are occupied by valence neutrons. If the valence neutrons occupy the  $\pi$  orbital, they retain two  $\alpha$  clusters in an inner region to gain potential energy. On the other hand, if the valence neutrons occupy the  $\sigma$  orbital, two  $\alpha$  clusters are pushed outward, because the  $\sigma$  orbital has two nodes along the  $\alpha$ - $\alpha$  direction, thus gaining kinetic energy as the  $\alpha$ - $\alpha$  distance increases. This lowering mechanism of the  $\sigma$  orbital derives the  $\sigma$  orbital configuration into the lower energy region in the developed cluster system. Consequently, the level inversion occurs between the normal  $\pi$  orbital and the higher nodal  $\sigma$  orbital and the  $N = 8$  magic number breaks down in very neutron rich Be such as  $^{11}\text{Be}$  and  $^{12}\text{Be}$ . According to the AMD calculations, it is found that the level inversion (i.e., the breaking of the neutron magic number  $N = 8$ ) occurs in  $^{12}\text{Be}$  and  $^{13}\text{Be}$  as well as in  $^{11}\text{Be}$ . For

these nuclei, largely deformed ground states having the highly developed clustering are obtained.

The theoretically predicted large deformation is consistent with the experimental reports on the strong  $E2$  transitions in the ground band [46–48]. The breaking of the neutron magicity in  $^{12}\text{Be}$  has been more directly evidenced by the intruder configuration in the ground state measured by  $1n$ -knockout reactions, which has been experimentally observed [49, 50]. Moreover, the systematics of the charge radii of neutron-rich Be, which have been recently measured precisely, indicate the vanishing of the neutron magicity at  $N = 8$ . The charge radius is smallest in  $^{10}\text{Be}$  and it increases in  $^{11}\text{Be}$  and  $^{12}\text{Be}$  in the chain of Be isotopes. This means that the  $N$  dependence of the charge radii shows a kink, not at  $N = 8$ , but at  $N = 6$ . This may indicate that the neutron magic number at  $N = 8$  disappears or shifts to  $N = 6$ .

## III. CLUSTER RESONANCES IN HIGHLY EXCITED STATES OF NEUTRON-RICH NUCLEI

In highly excited states of neutron-rich Be isotopes, two-body cluster resonances containing neutron-rich He, such as  $^6\text{He}$  and  $^8\text{He}$  clusters, are expected to appear. For instance, He+He resonances in  $^{12}\text{Be}$  have been observed in  $^6\text{He}+^6\text{He}$  and  $^8\text{He}+^4\text{He}$  break-up reactions [16, 17, 23]. According to recent experimental and theoretical studies of  $^{10}\text{Be}$ ,  $^6\text{He}+^4\text{He}$  cluster resonances appear a few MeV higher than the  $^{10}\text{Be}(0_2^+)$  of the MO bond structure [51–53]. These weakly coupling cluster states differ from the strongly coupling cluster states of the MO bond structure as mentioned before.

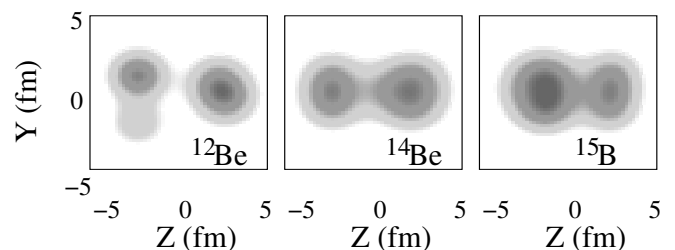


Fig. 1. Density distribution of  $^6\text{He}+^6\text{He}$ ,  $^6\text{He}+^8\text{He}$ , and  $^6\text{He}+^9\text{Li}$  cluster states in  $^{12}\text{Be}$ ,  $^{14}\text{Be}$ , and  $^{15}\text{B}$ . These states are obtained in the energy region near the corresponding threshold energy with the AMD+VAP calculation using the modified Volkov interaction supplemented by the spin-orbit force [12].

Moreover, various cluster resonances containing exotic clusters that are unstable nuclei themselves were theoretically predicted in neutron-rich nuclei. As an example, we obtain the  $^6\text{He}$  and  $t$  cluster resonances in  $^9\text{Li}$  with the theoretical calculation. Also in  $^{14}\text{Be}$  and  $^{15}\text{B}$ ,  $^8\text{He}+^6\text{He}$  and  $^9\text{Li}+^6\text{He}$  cluster structures were obtained in highly excited states [12, 52] (see Fig. 1). These cluster resonances are expected in the energy region near the corresponding threshold energy. Further experiments should search for those new cluster resonances near or above the threshold energy in neutron-rich nuclei.

The systematic study of cluster structures of excited states in unstable nuclei is requested to obtain a new energy rule for cluster states in unstable nuclei as Ikeda's threshold rule for cluster states in stable nuclei [54].

#### IV. SUMMARY

Cluster structures in light unstable nuclei were discussed. The structures of neutron-rich Be isotopes were theoretically investigated and the molecular orbital bond structure and its role in the vanishing of the neutron magic number  $N = 8$  were discussed. The two-body cluster resonances were predicted in highly excited states of neutron-rich Li, Be and B isotopes.

The systematic study of cluster structures has revealed that

cluster is one of the essential features of nuclear systems and that cluster states and mean-field states coexist in low-energy levels. The cluster feature is remarkable in particular in low-density systems which are realized in excited states near the threshold energy. This cluster enhancement in low density is the common feature not only in nuclear structure but also in heavy ion collision and infinite nuclear matter at finite temperature as known in the phenomena of multifragmentation and nuclear pasta formation in a neutron star.

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